Positivity and Slope Limiters for Lax-Wendroff Discontinuous Galerkin Methods

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Abstract

Hyperbolic conservation laws model phenomena characterized by waves propagating at finite speeds. A feature of such equations is that smooth initial data can become discontinuous in finite time – such discontinuities are referred to as shock waves. In recent years, the discontinuous Galerkin (DG) method has become one of the standard approaches for obtaining numerical solutions to hyperbolic conservation laws. For sufficiently smooth solutions, DG methods can be made arbitrarily high-order. However, when the solution becomes discontinuous (i.e., shock waves), DG methods can produce unphysical oscillations that result in large errors, numerical instabilities, and unphysical states such as negative densities. In this work my REU group studied the so-called Lax-Wendroff variant of DG. We formulate the method as a locally-implicit predictor step followed by an explicit corrector step. We develop a novel limiting strategy that works on both these steps and suppresses unphysical oscillations and maintains the positivity of key physical quantities. The resulting method is implemented in a Python code that we are making freely available and applied to several standard one-dimensional test problems.